OCR
Oxford Cambridge and RSA

Wednesday 22 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

1 Solve the equation $4x^{-\frac{1}{2}} = 7$, giving your answer as a fraction in its lowest terms. **[3]**

$4x^{-1/2} = 7 \rightarrow \frac{4}{\sqrt{x}} = 7$

$\rightarrow \frac{4}{7} = \sqrt{x} \rightarrow x = \frac{4^2}{7^2} = \frac{16}{49}$

$\therefore x = \frac{16}{49}$

2 Fig. 2 shows a triangle with one angle of 117° given. The lengths are given in centimetres.

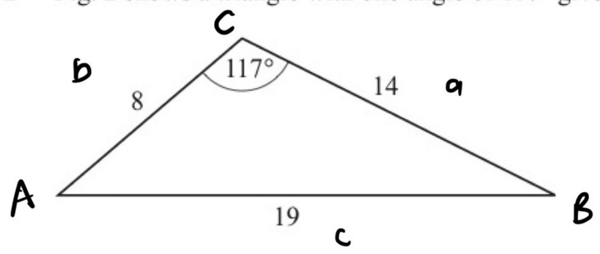


Fig. 2

Calculate the area of the triangle, giving your answer correct to 3 significant figures. **[2]**

$\text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(14)(8)\sin(117) = 49.8963\ldots = 49.9 \text{ cm}^2$ (3sf)

3 **Without** using a calculator, prove that $3\sqrt{2} > 2\sqrt{3}$. [3]

1 $3\sqrt{2} = \sqrt{3^2 \times 2} = \sqrt{9 \times 2} = \sqrt{18}$

2 $2\sqrt{3} = \sqrt{2^2 \times 3} = \sqrt{4 \times 3} = \sqrt{12}$

$\sqrt{18} > \sqrt{12} \quad \therefore \quad 3\sqrt{2} > 2\sqrt{3}$

4 The equation of a circle is $x^2 + y^2 + 8x - 6y - 39 = 0$.

(a) Find the coordinates of the centre of the circle. [2]

(b) Find the radius of the circle. [1]

a) $(x^2 + 8x) + (y^2 - 6y) - 39 = 0$

$(x + 4)^2 - 4^2 + (y - 3)^2 - (-3)^2 - 39 = 0 \rightarrow$ centre $= (-4, 3)$

b) $(x + 4)^2 - 16 + (y - 3)^2 - 9 - 39 = 0$

$(x + 4)^2 + (y - 3)^2 - 25 - 39 = 0 \rightarrow (x + 4)^2 + (y - 3)^2 = 64 \rightarrow$ as $(x - a)^2 + (y - b)^2 = r^2$

$64 = r^2 \therefore r = 8$

$(x - a)^2 + (y - b)^2 = r^2$ is the general equation of a circle

5 Each day John either cycles to work or goes on the bus.

- If it is raining when John is ready to set off for work, the probability that he cycles to work is 0.4.
- If it is not raining when John is ready to set off for work, the probability that he cycles to work is 0.9.
- The probability that it is raining when he is ready to set off for work is 0.2.

You should assume that days on which it rains occur randomly and independently.

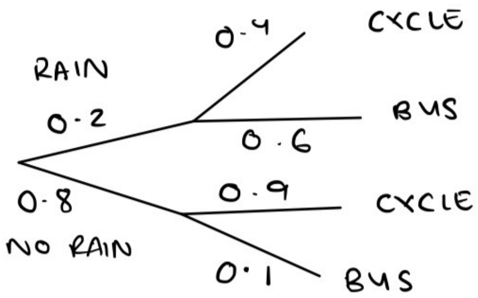
(a) Draw a tree diagram to show the possible outcomes and their associated probabilities. **[3]**

(b) Calculate the probability that, on a randomly chosen day, John cycles to work. **[3]**

John works 5 days each week.

(c) Calculate the probability that he cycles to work every day in a randomly chosen working week. **[2]**

a)



b) P (cycles) $= (P(R) \times P(C)) + (P(NR) \times P(C))$

$= (0.2 \times 0.4) + (0.8 \times 0.9) = 0.08 + 0.72$

$= \underline{\underline{0.8}}$

c) He works 5 days:

P(Cycles every day) $= (0.8)^5 = 0.32768 \approx \underline{\underline{0.328}}$ (3 sf)

6 The large data set gives information about life expectancy at birth for males and females in different London boroughs. Fig. 6.1 shows summary statistics for female life expectancy at birth for the years 2012–2014. Fig. 6.2 shows summary statistics for male life expectancy at birth for the years 2012–2014.

Female Life Expectancy at Birth

n	32
Mean	84.2313
s	1.1563
$\sum x$	2695.4
$\sum x^2$	227078.36
Min	82.1
Q1	83.45
Median	84
Q3	84.9
Max	86.7

Fig. 6.1

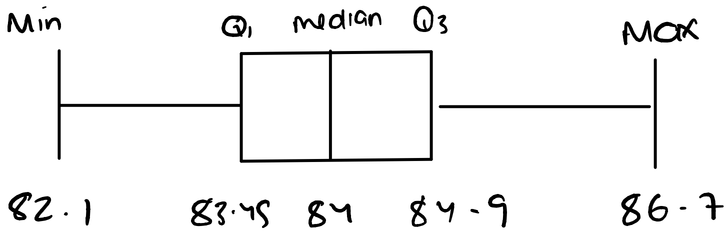
Male Life Expectancy at Birth

n	32
Mean	80.2844
s	1.4294
$\sum x$	2569.1
$\sum x^2$	206321.93
Min	77.6
Q1	79
Median	80.25
Q3	81.15
Max	83.3

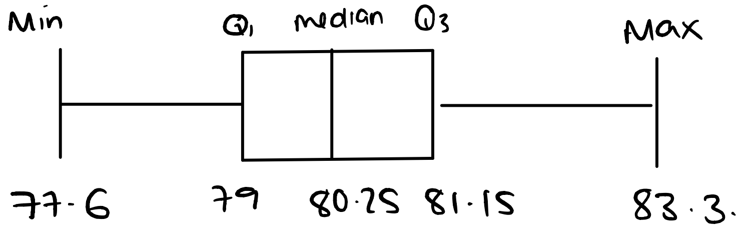
Fig. 6.2

(a) Use the information in Fig. 6.1 and Fig. 6.2 to draw two box plots. Draw one box plot for female life expectancy at birth in London boroughs and one box plot for male life expectancy at birth in London boroughs. **[5]**

female life expectancy



male life expectancy



(b) Compare and contrast the distribution of male life expectancy at birth with the distribution of female life expectancy at birth in London boroughs in 2012–2014. **[2]**

1. $80.25 < 84$ therefore the average male life expectancy is lower than the average female life expectancy
2. The interquartile range for men $(81.15 - 79 = 2.15)$ is larger than the interquartile range for women $(84.9 - 83.45 = 1.45)$, so male life expectancy is more variable.

Lorraine, who lives in Lancashire, says she wishes her daughter (who was born in 2013) had been born in the London borough of Barnet, because her daughter would have had a higher life expectancy.

(c) Give **two** reasons why there is no evidence in the large data set to support Lorraine's comment. **[2]**

1. There is no data for Lancashire to compare to.
2. The data set doesn't definitely show the life expectancy for one person

(d) Use the mean and standard deviation for the summary statistics given in Fig. 6.1 and Fig. 6.2 to show that there is at least one outlier in each set. **[2]**

female life expectancy:

$84.2313 + (2 \times 1.1563) = 86.5439$

$84.2313 - (2 \times 1.1563) = 81.9187$

$86.7 > 86.5439$ ∴ outlier

(max point)

male life expectancy:

$80.2844 + (2 \times 1.4294) = 83.1432$

$80.2844 - (2 \times 1.4294) = 77.4256$

$83.3 > 83.1432$ ∴ outlier

(max point)

FORMULA: mean $\pm (2 \times s.d.)$

The scatter diagram in Fig. 6.3 shows male life expectancy at birth plotted against female life expectancy at birth for London boroughs in 2012–14. The outliers have been removed.

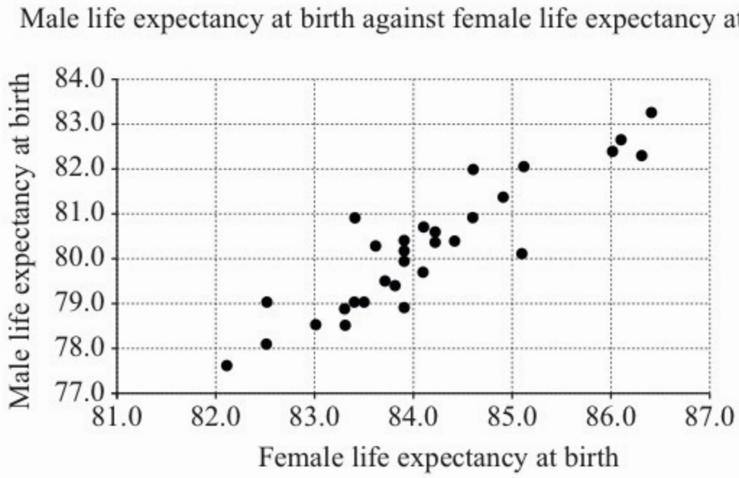


Fig. 6.3

(e) Describe the association between male life expectancy at birth and female life expectancy at birth in London boroughs in 2012–14. **[2]**

There is a positive correlation between male and female life expectancy at birth in London boroughs in 2012-14.

7 **(a)** Find $\int x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)dx$. **[5]**

(b) Show that $\int_0^8 x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)dx = a\times 2^{11}$, where a is a positive integer to be determined. **[3]**

a) $\int x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)dx = \int 15x^4 + \frac{11x^3}{x^{1/3}}\,dx = \int 15x^4 + \frac{11x^{9/3}}{x^{1/3}}\,dx = \int 15x^4 + 11x^{8/3}\,dx$

$\rightarrow \int 15x^4\,dx + \int 11x^{8/3}\,dx$

$= \frac{15}{5}x^5 + \frac{11\times 3}{11}x^{11/3} + C = \underline{\underline{3x^5 + 3x^{11/3} + C}}$ → don't forget the "+ C"!

b) $\int_0^8 x^3\left(15x+\frac{11}{\sqrt[3]{x}}\right)dx = \left[3x^5 + 3x^{11/3}\right]_0^8 = \left[3(8^5)+3(8^{11/3})\right] - [0]$

$= 3(8^5) + 3(\sqrt[3]{8})^{11} = 3(2^3)^5 + 3(2^{11})$

$= 3(2^{15}) + 3(2^{11})$

$= (3(2^4)+3)\times 2^{11}$

$= 51\times 2^{11}$ where $a = 51$

REMEMBER:

$(x^a)^b = x^{ab}$

$x^a \times x^b = x^{a+b}$

8 According to the latest research there are 19.8 million male drivers and 16.2 million female drivers on the roads in the UK.

(a) A driver in the UK is selected at random. Find the probability that the driver is male. **[1]**

(b) Calculate the probability that there are 7 female drivers in a random sample of 25 UK drivers. **[1]**

a) $\frac{19.8 \times 10^6}{(19.8 + 16.2) \times 10^6} = \frac{19.8}{36} = 0.55 \rightarrow P(\text{male}) = 0.55$

b) $P(\text{female}) = 0.45$
$X \sim B(25, 0.45)$ | $P(X = 7) = 0.03809\ldots$
$= 0.0381$ (3sf)

When driving in a built-up area, Rebecca exceeded the speed limit and was obliged to attend a speed awareness course. Her husband said "It's nearly always male drivers who are speeding." When Rebecca attends the course, she finds that there are 25 drivers, 7 of whom are female. You should assume that the drivers on the speed awareness course constitute a random sample of drivers caught speeding.

(c) **In this question you must show detailed reasoning.**

Conduct a hypothesis test to determine whether there is any evidence at the 5% level to suggest that male drivers are more likely to exceed the speed limit than female drivers. **[7]**

(d) State a modelling assumption that is necessary in order to conduct the hypothesis test in part **(c)**. **[1]**

c) $H_0 : p = 0.55$ p is the probability that a
$H_1 : p > 0.55$ driver speeding is male

$X \sim B(25, 0.45)$

$P(X \geq 18) = 1 - P(X \leq 17)$
$= 1 - 0.936149 = 0.06385$

at 5% sig level

$0.06385 > 0.05$ $\therefore$ the result is insignificant.

There is insufficient evidence to reject Ho, would suggest that female drivers are more likely to be speeding.

d) The probabity of a driver getting caught speeding is independent of any other driver caught speeding.

9 In 2012 Adam bought a second hand car for £8500. Each year Adam has his car valued. He believes that there is a non-linear relationship between t, the time in years since he bought the car, and V, the value of the car in pounds. Fig. 9.1 shows successive values of V and $\log_{10}V$.

t	0	1	2	3	4
V	8500	6970	5720	4690	3840
$\log_{10}V$	3.93	3.84	3.76	3.67	3.58

Fig. 9.1

Adam uses a spreadsheet to plot the points $(t, \log_{10}V)$ shown in Fig. 9.1, and then generates a line of best fit for these points. The line passes through the points (0, 3.93) and (4, 3.58). A copy of his graph is shown in Fig. 9.2.

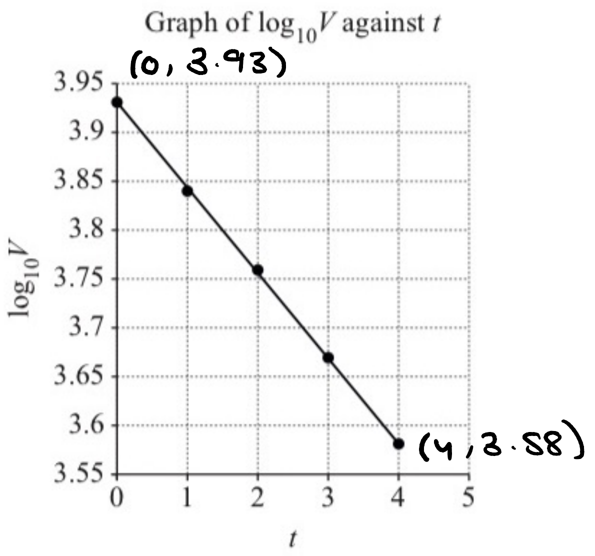


Fig. 9.2

(a) Find an expression for $\log_{10}V$ in terms of t. → $\boxed{\text{in the form } y = mx + c}$ **[3]**

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.58 - 3.93}{4 - 0} = \frac{-0.35}{4} = -0.0875$

y-intercept $= 3.93$

$\therefore \log_{10}V = -0.0875t + 3.93$

(b) Find a model for V in the form $V = A \times b^t$, where A and b are constants to be determined. Give the values of A and b correct to 2 significant figures. **[3]**

$\log_{10}V = -0.0875t \times 3.93$

$V = 10^{-0.0875t} \times 10^{3.93} \rightarrow V = 0.81752..^{t} \times 8511.38...$

$V = 0.82^t \times 8500$ (2sf)

where $A = 8500$ and b is 0.82

In 2017 Adam's car was valued at £3150.

(c) Determine whether the model is a good fit for this data. **[1]**

$2017 - 2012 = 5$

$V = 0.82^5 \times 8500 = 3151.288 \approx 3150$

$\therefore$ the model is good fit

A company called Webuyoldcars pays £500 for any second hand car. Adam decides that he will sell his car to this company when the annual valuation of his car is less than £500.

(d) According to the model, after how many years will Adam sell his car to Webuyoldcars? **[3]**

$500 = 0.82^t \times 8500 \rightarrow \frac{500}{8500} = 0.82^t \rightarrow \log\left(\frac{500}{8500}\right) = t\log(0.82)$

$= \frac{-1.23044892138}{\log(0.82)} = t = 14.27664$

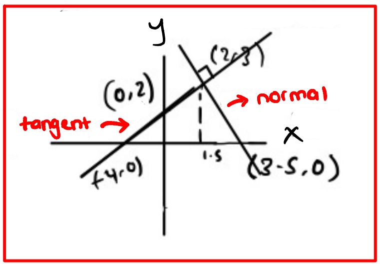
$\therefore$ after 15 years

Don't get caught out with 14 years. the 0.27... also counts

10 In this question you must show detailed reasoning.

The equation of a curve is $y = \frac{x^2}{4} + \frac{2}{x} + 1$. A tangent and a normal to the curve are drawn at the point where $x = 2$.

Calculate the area bounded by the tangent, the normal and the x-axis. **[10]**



← A DIAGRAM ALLOWS YOU TO SEE WHAT FORMULA FOR AREA YOU NEED.

$y = \frac{x^2}{4} + \frac{2}{x} + 1 = \frac{1}{4}x^2 + 2x^{-1} + 1$

when $x = 2$:

$y = \frac{1}{4}(2^2) + 2(2^{-1}) + 1 = \frac{4}{4} + \frac{2}{2} + 1 = 3$

[gives the point of intersection]

$x = 2,\ y = 3$

$\frac{dy}{dx} = \frac{1}{2}x - 2x^{-2} \Rightarrow x = 2$

gradient: $\frac{2}{2} - \frac{2}{2^2} = 1 - \frac{2}{4} = \frac{1}{2}$

Equation of the tangent:

$y - 3 = \frac{1}{2}(x-2) \rightarrow y - 3 = \frac{x}{2} - 1 \rightarrow y = \frac{x}{2} + 2$

x-intercept

$-2 = \frac{x}{2}$

$x = -4$

$\therefore (-4, 0)$

Equation of the normal:

gradient $= -2$ so $y - 3 = -2(x-2)$

$y - 3 = -2x + 4$

$y = -2x + 7$

x-intercept

$2x = 7$

$x = 3.5$

$\therefore (3.5, 0)$

Area:

$\left(\frac{1}{2} \times 2 \times 3\right) + \left(\frac{1}{2} \times (1.5 + 4) \times 3\right)$

$= \left(\frac{6}{2}\right) + \left(\frac{16.5}{2}\right) = \frac{22.5}{2} = \frac{45}{4}$

$\therefore$ area bounded is $\frac{45}{4}$